UUCMS NO

B.M.S COLLEGE FOR WOMEN AUTONOMOUS

BENGALURU -560004

END SEMESTER EXAMINATION – APRIL/ MAY 2023

M.Sc. Mathematics – I Semester

TOPOLOGY – I

Course Code MM103T **QP Code: 11003**

Duration : 3 Hours **Maximum Marks: 70**

Instructions: 1) **All** questions carry **equal** marks.
2) Answer **any five** questions

1. a) Define an infinite set. Let X be an infinite set and let $x_0 \in X$, then prove that $X - \{x_0\}$ is infinite.
   b) Prove that a set X is finite if and only if either $X = \emptyset$ or X is in one-to-one correspondence with some $\mathbb{N}_k = \{1,2, \dots k\}$ set of all natural numbers from 1 to k.

   (6 + 8)

2. a) Prove that set of real numbers is uncountable.
   b) Prove that for any set X, $\overline{\overline{P(X)}} = 2^{\bar{\bar{X}}}$.
   c) With usual notations, prove that $2^{\aleph_0} = c$.

   (6 + 5 + 4)

3. a) Define metric space. Suppose (X, d) is a metric space, let $d_1(x, y) = \frac{d(x,y)}{1+d(x,y)}$ be defined on $X \times X$. Prove that d_1 is a metric on X.
   b) Prove that every closed subspace of a complete metric space is complete.

   (8 + 6)

4. a) State and prove Cantor's intersection theorem.
   b) Prove that every complete metric space is of the second category.

   (7 + 7)

5. a) Define topological space. Prove that every metric space is a topological space.
   b) Define (i) neighbourhood of a point (ii) limit point of a set. Let $A \subseteq (X, \mathcal{T})$ then prove that $A \cup D(A)$ is closed, where $D(A)$ is derived set of A.

   (7 + 7)

6. a) Define interior and boundary of a set. Prove that $(A')^0 = (\bar{A})'$.
   b) Prove that $\bar{A} = A \cup b(A)$.

   (8 + 6)

7. a) Given a function $f: X \to Y$, show that f is continuous if and only if the inverses of every closed set is closed.

   b) Define homeomorphism. Show that a bijective function $f: X \to Y$ is a homeomorphism if and only if $f(\bar{A}) = \overline{f(A)}$, $\forall A \subseteq X$.

(7 + 7)

8. a) Prove that the following are equivalent

   i. X is connected.

   ii. X and $\emptyset$ are the only sets which are both open and closed.

   iii. X is not the union of two non-empty disjoint open sets.

   iv. X is not the union of two non-empty disjoint closed sets.

   b) Let C be a connected subset of $(X, \mathcal{T})$ where X has a separation $X = A \cup B$, then prove that $C \subseteq A$ or $C \subseteq B$. **(6 + 8)**

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